# Quantum Mechanics
# Sem -III

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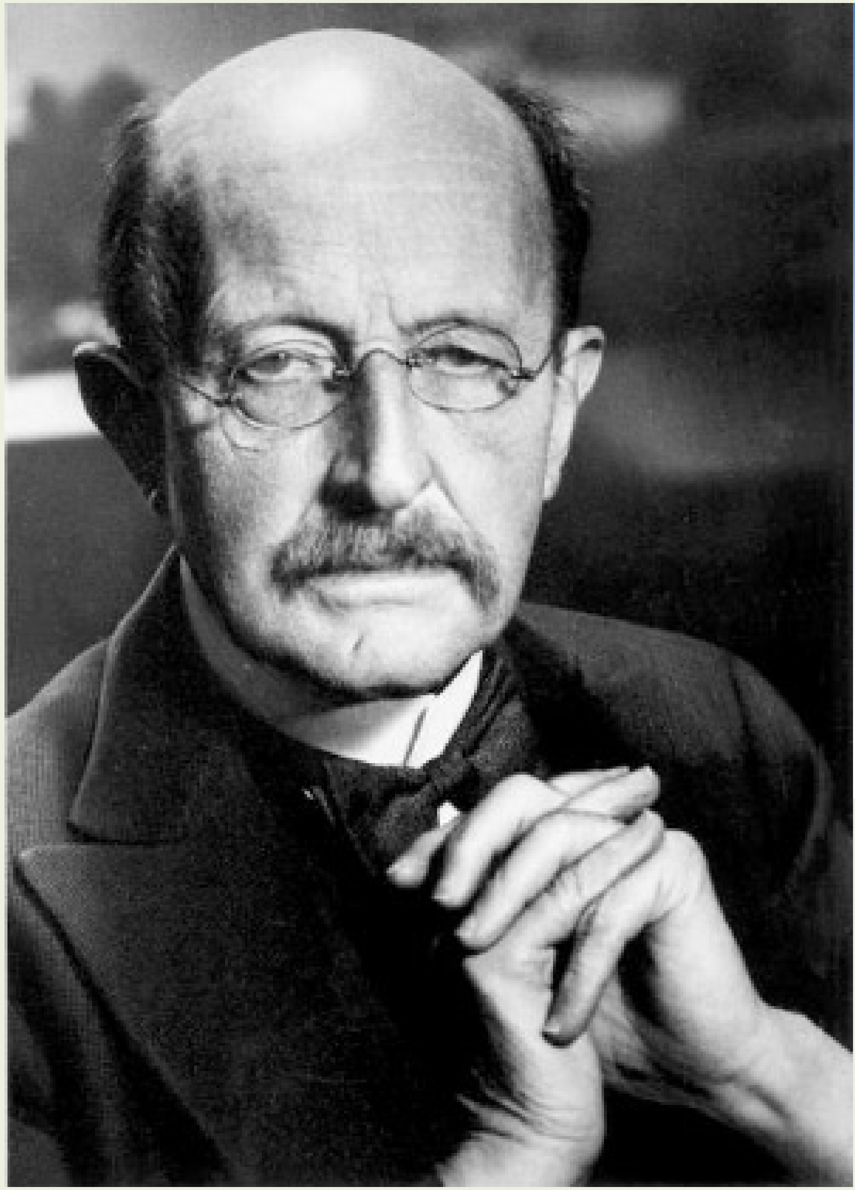


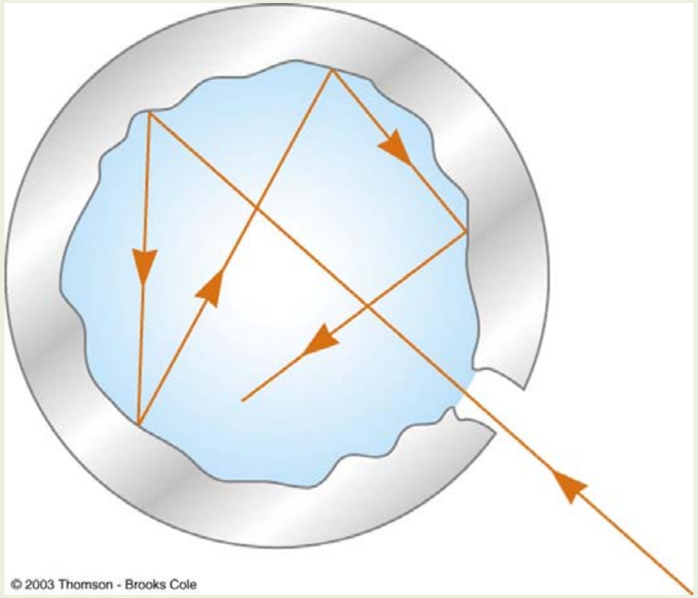
**Max Karl Ernst Ludwig Planck**
**1858-1947**

# Birthday of Quantum Physics on 14th December, 1900

$$E = h\nu$$

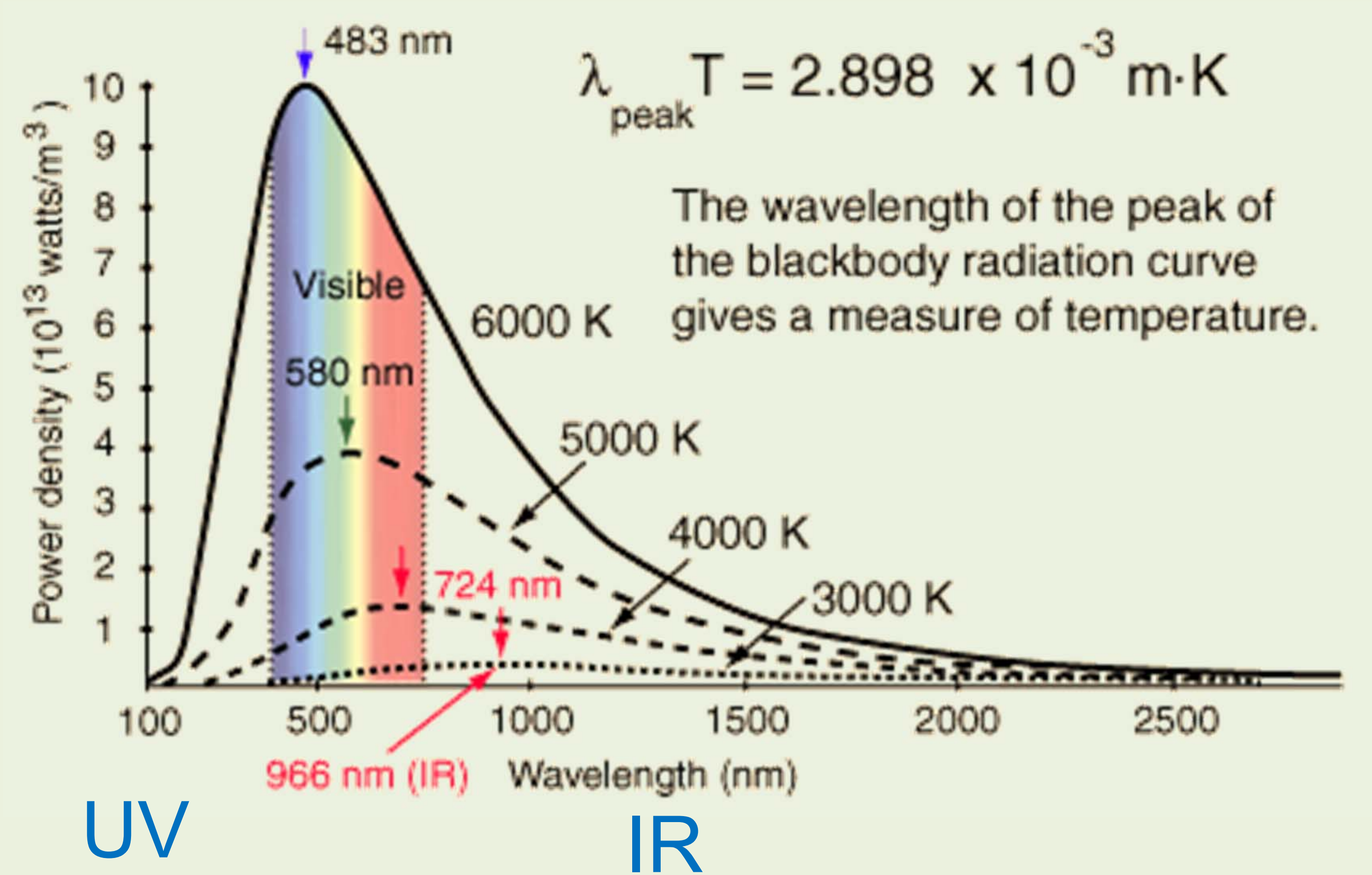
Planck introduces a new fundamental constant **h** to explain black-body radiation

# Blackbody Radiation



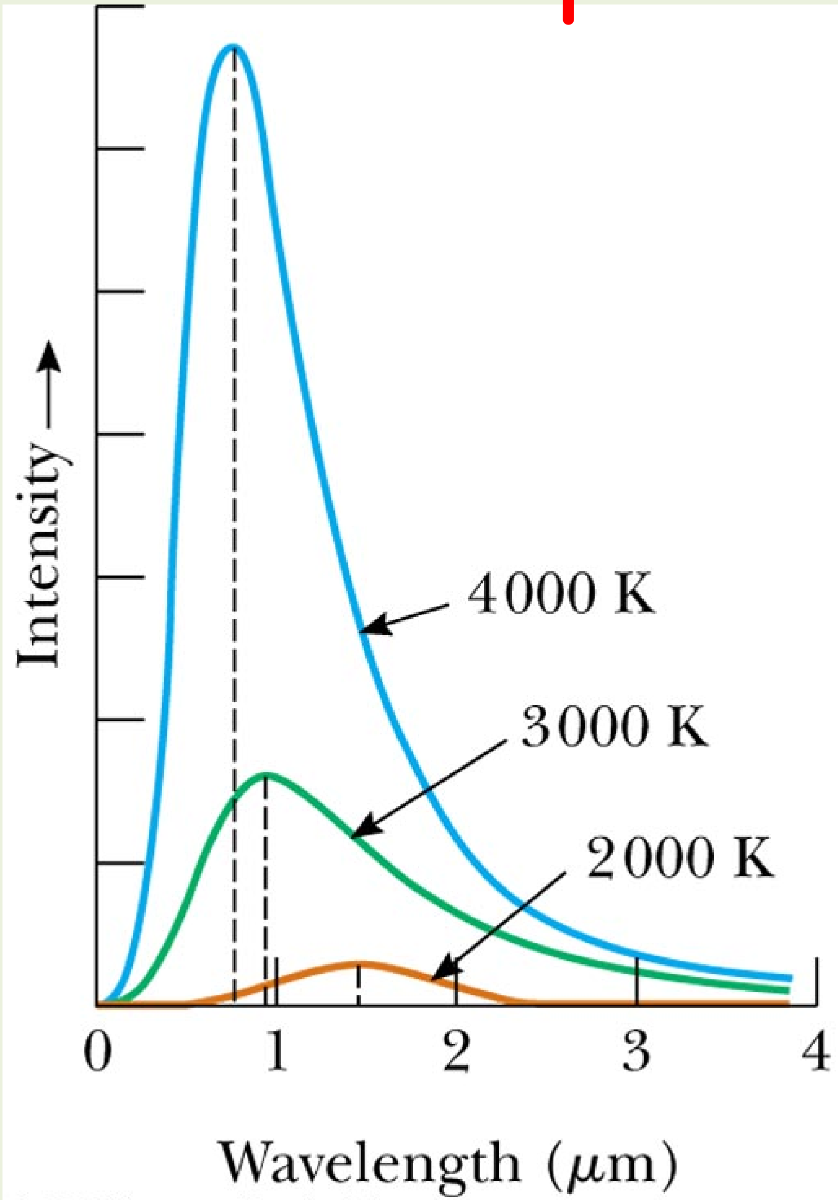

Blackbody Radiation: If a heated body emits electromagnetic radiation, which is not dependent of the chemical composition of the body, but depends only upon its temperature then the body is said to be a perfectly blackbody and radiation as blackbody radiation.

# Blackbody Radiation Graph

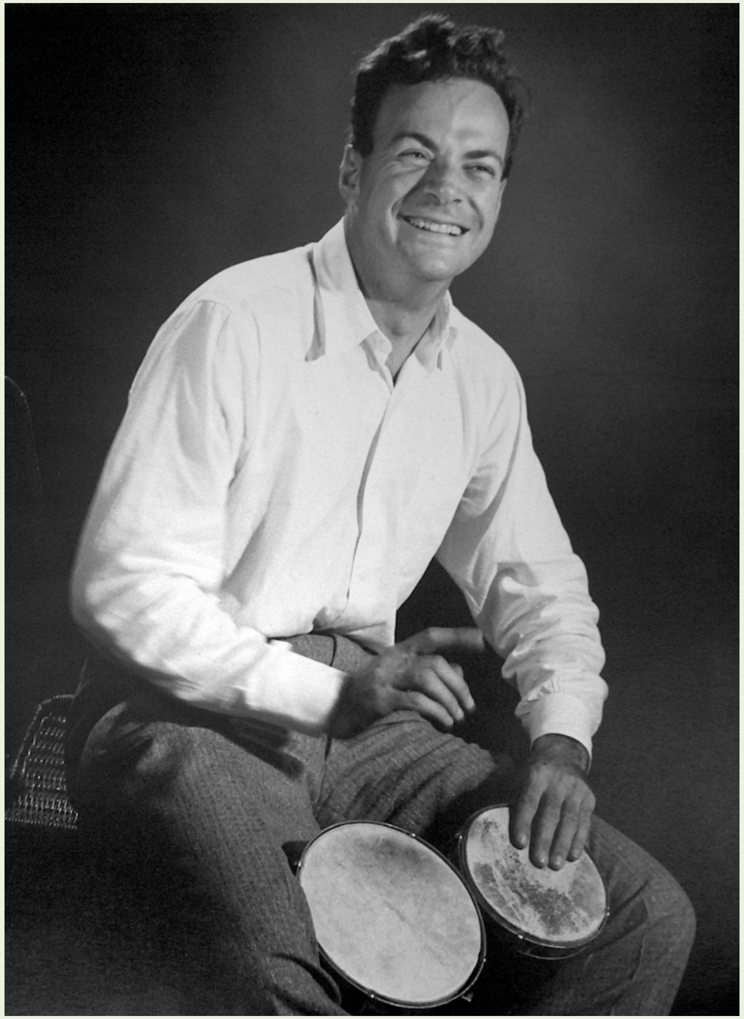


# Blackbody Radiation Graph

- Experimental data for distribution of energy in blackbody radiation
- As the temperature increases, the total amount of energy increases
  - Shown by the area under the curve
- As the temperature increases, the peak of the distribution shifts to shorter wavelengths



# Opinions on quantum mechanics



**Richard Feynman**
**(1918-1988)**

*I think it is safe to say that no one understands quantum mechanics. Do not keep saying to yourself, if you can possibly avoid it, "But how can it be like that?" because you will get "down the drain" into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that.*

- Richard Feynman

*Those who are not shocked when they first come across quantum mechanics cannot possibly have understood it.*

- Niels Bohr

# WAVE-PARTICLE DUALITY OF LIGHT

In 1924 Einstein wrote:- " There are therefore now two theories of light, both indispensable, and … without any logical connection."

Evidence for wave-nature of light

- Diffraction and interference
- Reflection, Refraction, Polarization

Evidence for particle-nature of light

- Photoelectric effect
- Compton effect

- Light exhibits diffraction and interference phenomena that are *only* explicable in terms of wave properties
- Light is always detected as packets (photons); if we look, we never observe half a photon
- Number of photons proportional to energy density (i.e. to square of electromagnetic field strength)

# Quantum Mechanics

- At the turn of the last century, there were several experimental observations which could not be explained by the established laws of classical physics and called for a radically different way of thinking.
- This led to the development of Quantum Mechanics which is today regarded as the fundamental theory of Nature.
- Quantum Mechanics is the most accurate and complete description of the physical world.
- It also forms a basis for the understanding of quantum information.

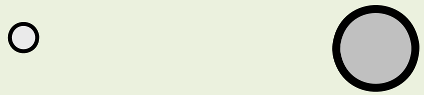
# Some key events/observations that led to the development of quantum mechanics…

- Black body radiation spectrum (Planck, 1901)
- Photoelectric effect (Einstein, 1905)
- Model of the atom (Rutherford, 1911)
- Quantum Theory of Spectra (Bohr, 1913)
- Scattering of photons by electrons (Compton, 1922)
- Exclusion Principle (Pauli, 1922)
- Matter Waves (de Broglie 1925)
- Experimental test of matter waves (Davisson and Germer, 1927)

# Particles and Waves

Two ways in which energy is transported:

Point mass interaction: transfers of momentum and kinetic energy: ***particles***.



Extended regions wherein energy is transferred by vibrations and rotations: ***waves***.

## Classical concepts never allow to think that

1. Wave may also behave like particle.
   (Planck's hypothesis)
2. Particle may behave like wave.
   (de Broglie hypothesis)
3. Position and momentum of a particle cannot be measured accurately & simultaneously.
   (Heisenberg uncertainty principle)
4. Energy of wave is related with frequency and is quantised.

$$E = nh\nu$$

**These new concepts is basically quantum concepts**

# Classical concept

Two distinct categories :

1. Material body (**particle**)
   Newton's laws of motion

2. Electromagnetic field (**wave**)
   Maxwell's equation

Position and velocity (momentum) are precisely measurable

Spread over the space, amplitude gives energy/intensity, frequency is nothing but time periodicity of oscillator

Additionally
Laws of thermodynamics

$E = kT$

**Fundamental constants :**
1. velocity of light c
2. Avogadro Number N
3. Boltzman constant k
4. Unit of charge e

$E = mc^2$

Velocity << c : non-relativistic
Velocity comparable to c : relativistic

# De Broglie Waves

If a light-wave could also act like a particle, why shouldn’t matter-particles also act like waves?

- In his thesis in 1923, Prince Louis V. de Broglie suggested that mass particles should have wave properties similar to electromagnetic radiation.
- The energy can be written as:

$$h\nu = pc \quad h\cancel{\nu} = p\lambda\cancel{\nu}$$

- Thus the wavelength of a matter wave is called **the de Broglie wavelength**:

$$\lambda = \frac{h}{p}$$

Louis V. de Broglie
(1892-1987)

# Relation between particle and wave properties of light

Energy and frequency $E = h\nu$

Also have relation between momentum and wavelength

Relativistic formula relating energy and momentum $E^2 = p^2c^2 + m^2c^4$

For light $E = pc$ and $c = \lambda\nu$

$$p = \frac{h}{\lambda} = \frac{h\nu}{c}$$

Also commonly write these as

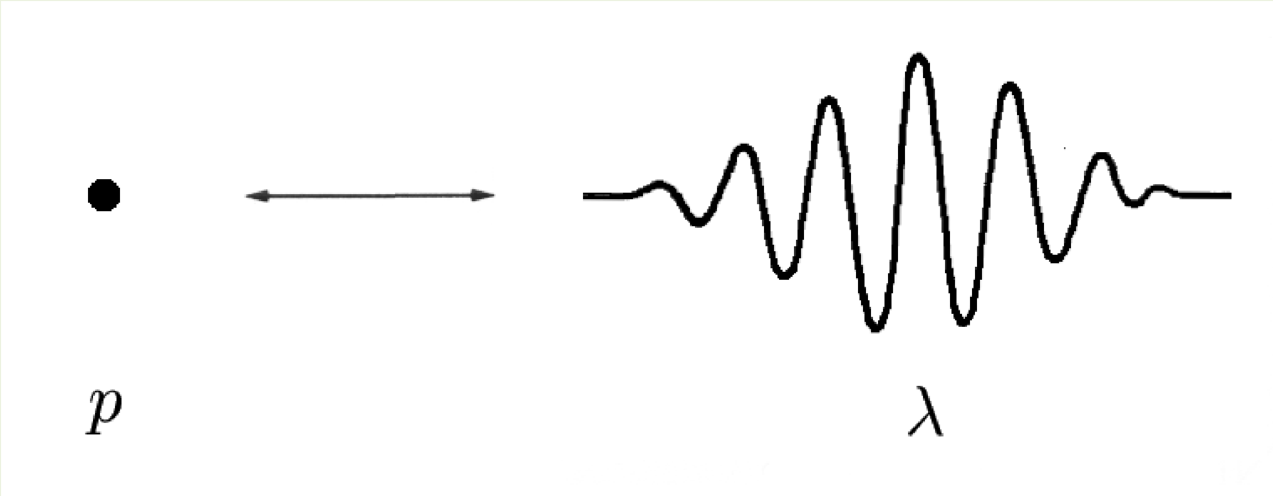
$E = \hbar\omega$ angular frequency $\omega = 2\pi\nu$

$p = \hbar k$ wavevector $k = \frac{2\pi}{\lambda}$

h bar $\hbar = \frac{h}{2\pi}$

# Wave-Particle Duality

particle wave function




$$\lambda = \frac{h}{p} = \frac{h}{m\,v}$$

# Example: de Broglie wavelength of a ball

- Mass = 1 kg
  Speed = 1 m / sec

$$\lambda = \frac{6.63 \times 10^{-34}\,\text{Joules} \cdot \text{sec}}{(1\,\text{kg})(1\,\text{m/sec})} = 6.63 \times 10^{-34}\,\text{m}$$

- *This is extremely small!* Thus, it is very difficult to observe the wave-like behaviour of ordinary objects.

# Example: de Broglie wavelength of an electron

- Mass = $9.11 \times 10^{-31}$ kg
  Speed = $10^{6}$ m / sec

$$\lambda = \frac{6.63 \times 10^{-34} \text{ Joules} \cdot \text{sec}}{(9.11 \times 10^{-31} \text{ kg})(10^{6} \text{ m/sec})} = 7.28 \times 10^{-10} \text{ m}$$

- This wavelength is in the region of
  X-rays

# Wave packet, phase velocity and group velocity

- The velocities of the individual waves which superpose to produce the wave packet representing the particle are different - the wave packet as a whole has a different velocity from the waves that comprise it

- Phase velocity: The rate at which the phase of the wave propagates in space

- Group velocity: The rate at which the envelope of the wave packet propagates

# Wave packet, phase velocity and group velocity

Phase velocity $$v_p = \frac{c^2}{v}$$

Group velocity $$v_g = v$$

Here $c$ is the velocity of light and $v$ is the velocity of the particle

# Wave Function

- Completely describes all the properties of a given particle

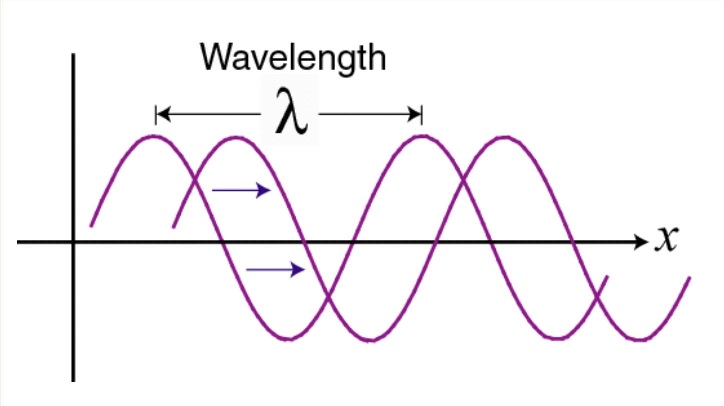
- Called $\psi = \psi\,(x,t)$; is a complex function of position $x$ and time $t$

- What is the meaning of this wave function?

# The Phase Velocity

How fast is the wave traveling?

Velocity is a reference distance divided by a reference time.




The phase velocity is the wavelength / period: $\mathrm{v} = \lambda / \tau$

Since $f = 1/\tau$:

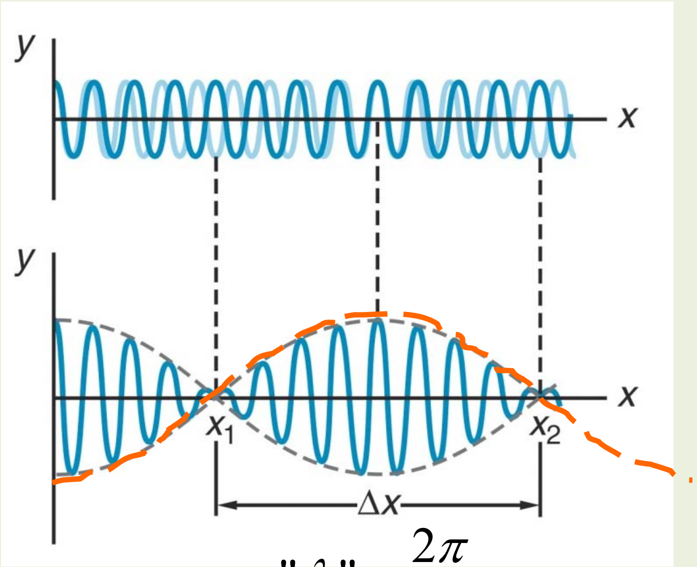
$$\mathrm{v} = \lambda f$$

In terms of k, $k = 2\pi / \lambda$, and
the angular frequency, $\omega = 2\pi / \tau$, this is:

$$\mathrm{v} = \omega / k$$

# Phase and Group Velocities

This ***phase*** velocity $v_p = \frac{\omega}{k} = \frac{E}{p} = \frac{\gamma mc^2}{\gamma mv} = \frac{c^2}{v} > c$ - no limitations on the phase velocity, (phase of a plane wave does not carry any information)




$$\leftarrow \text{"}\lambda\text{"} \approx \frac{2\pi}{\Delta k} \rightarrow$$

The observable is the ***group*** velocity (the velocity of propagation of a wave "packet" or wave "group". Let's consider the superposition of two harmonic waves with slightly different frequencies (ω>>Δω, k>>Δk):

$$y_1 = A\cos(\omega t - kx)$$

$$y_2 = A\cos[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$\cos\alpha + \cos\beta = 2\cos\left[\frac{1}{2}(\alpha+\beta)\right]\cos\left[\frac{1}{2}(\alpha-\beta)\right]$$

$$y = y_1 + y_2 = 2A\cos\left[\frac{1}{2}\{(2\omega + \Delta\omega)t - (2k + \Delta k)x\}\right]\cos\left[\frac{1}{2}(\Delta\omega \cdot t - \Delta k \cdot x)\right]$$

$$\approx \underbrace{2A\cos(\omega t - kx)}_{\text{fast oscillations within the wave group}}\underbrace{\cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)}_{\text{"envelope"= wave group}}$$

The velocity of propagation of the wave packet:

$$v_g = \frac{d\omega}{dk}$$

-the ***group*** velocity

# Group Velocity of de Broglie waves

$$v_g = \frac{d\omega}{dk} = \frac{dE}{dp} \qquad E = \sqrt{(pc)^2 + (mc^2)^2}$$

$$\boxed{v_g = v}$$

- the group velocity of de Broglie waves coincide with the particle's velocity

$$\frac{dE}{dp} = \frac{1}{2}\frac{2pc^2}{\sqrt{(pc)^2 + (mc^2)^2}} = \frac{pc^2}{E} = \frac{\gamma mv \cdot c^2}{\gamma mc^2} = v$$

$$\boxed{v_g v_p = c^2}$$

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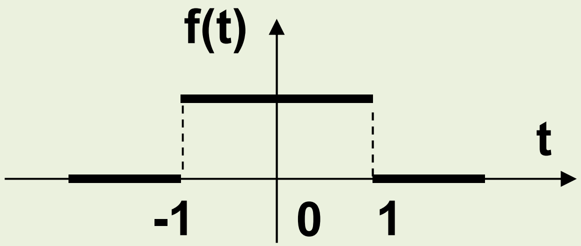
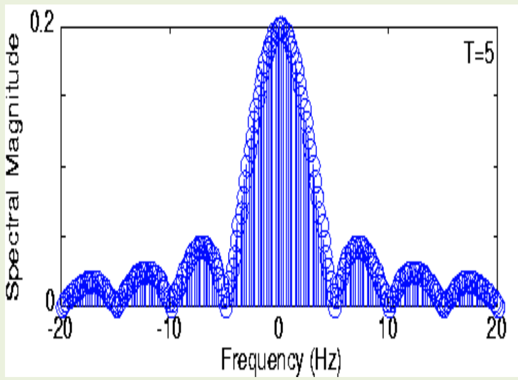
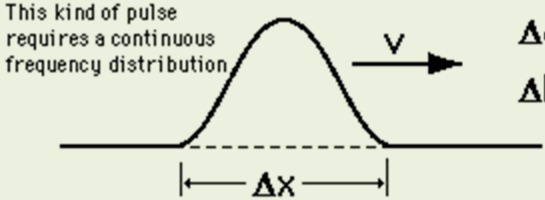
Periodic processes: discrete spectrum (Fourier series).

Aperiodic processes: continuous spectrum (represented as Fourier integral)



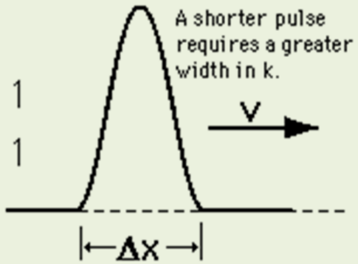




$$\mathrm{sinc}(\omega) = \begin{cases} 1, & \omega = 0 \\ \dfrac{\sin\omega}{\omega}, & \omega \neq 0 \end{cases}$$

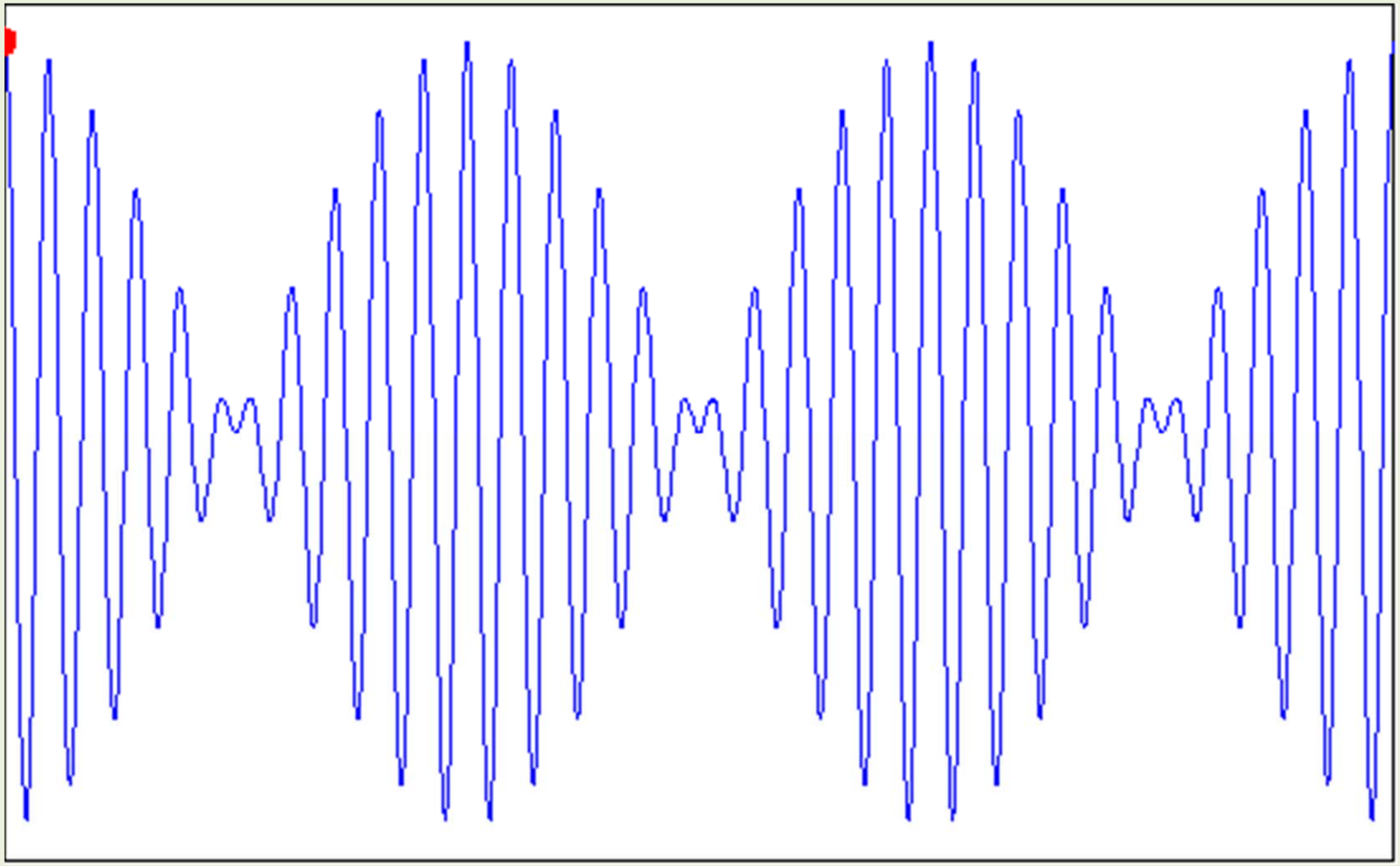



$\Delta\omega\Delta t \approx 1$

$\Delta k\Delta x \approx 1$



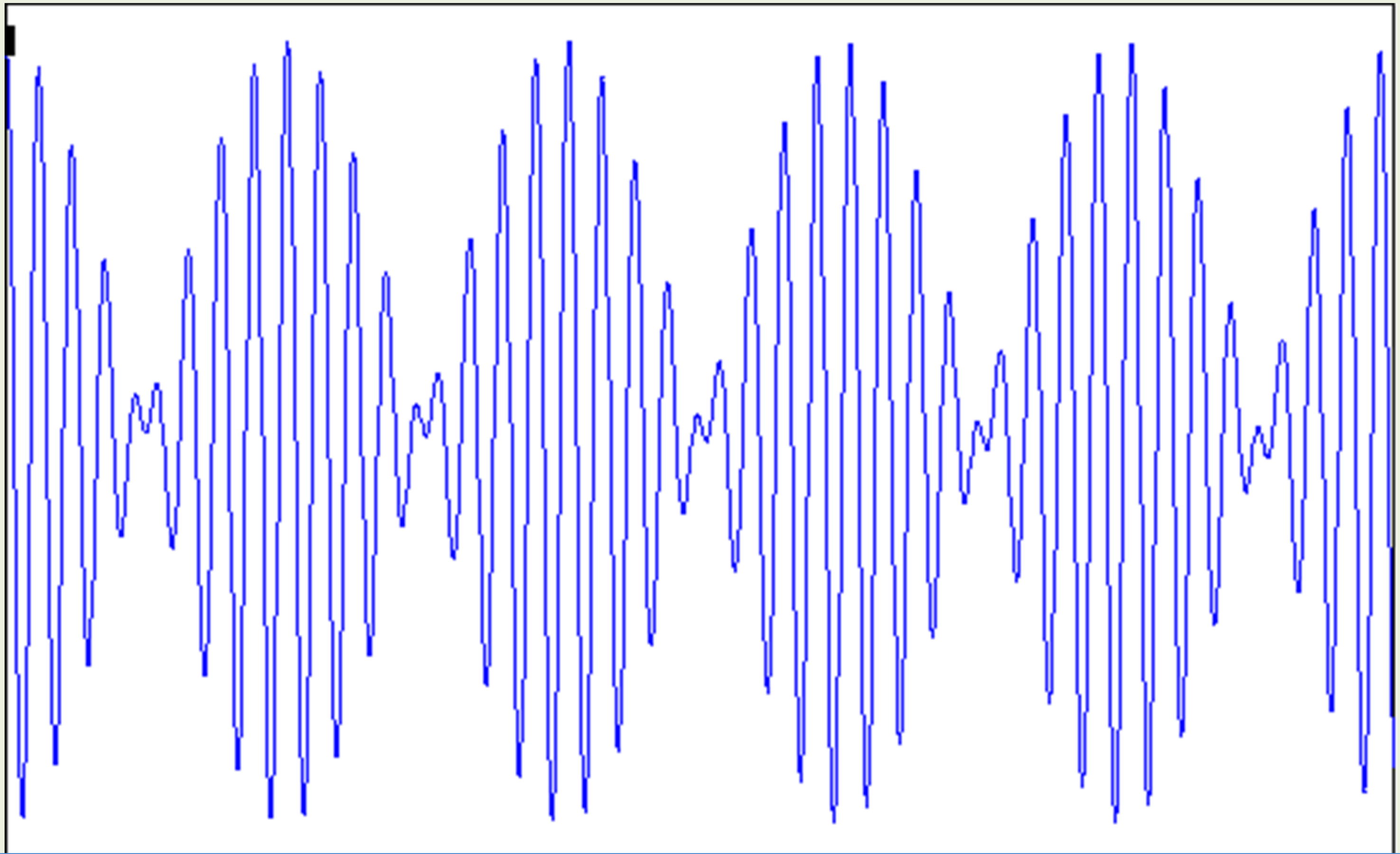
# The Group Velocity



This is the velocity at which the overall shape of the wave's amplitudes, or the wave 'envelope', propagates. (= *signal velocity*)

Here, phase velocity = group velocity (the medium is *non-dispersive*)

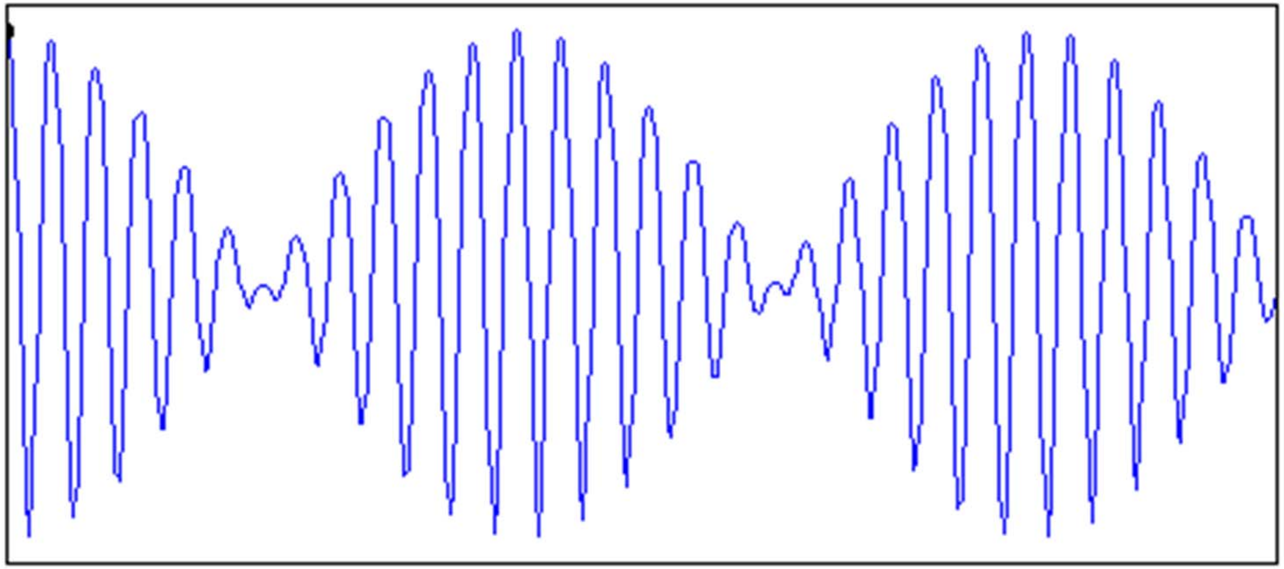
# Dispersion: phase/group velocity depends on frequency



Black dot moves at phase velocity. Red dot moves at group velocity.

This is *normal dispersion* (refractive index *decreases with increasing* $\lambda$)

# Dispersion: phase/group velocity depends on frequency

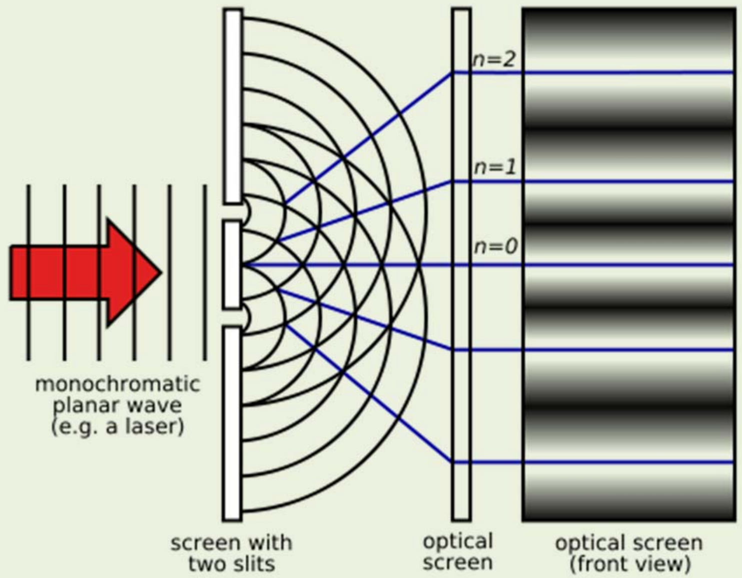


**Black dot moves at group velocity. Red dot moves at phase velocity.**

**This is *anomalous dispersion* (refractive index *increases with increasing λ*)**

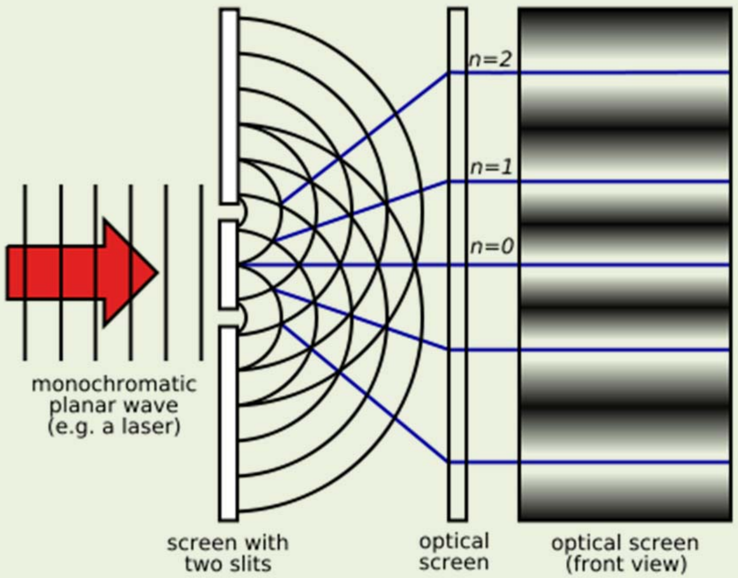
# Double-slit experiments:

## Light:

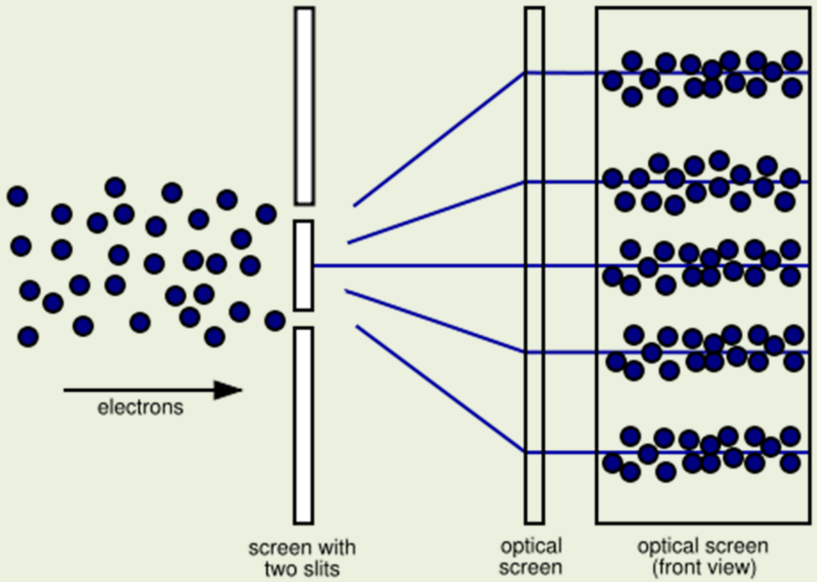


# Double-slit experiments:

## Light:




## Electrons:



# Individual electrons:

- In previous experiments many electrons were diffracted (or show interference)
- Will one get the same result for a single electron?
- Such experiments were performed
  - Intensity of the electron beam was so low that only one electron at a time proceeds
  - Still diffraction (and interference) patterns, and not diffused scattering, were observed, confirming that

Thus individual electrons possess wave properties!!!

# Complementarity

- Thus Heisenberg's principle says that quantum mechanics imposes certain limits on the accuracy with which we can observe the world.

- A pair of variables like position and its associated momentum which we cannot observe accurately together are said to be complementary variables.

- Other such complementary pairs are rotational angle and the associated angular momentum.

- The product of each pair has dimensions =[h]!

## Complimentarity Principle:

**The particle and the wave models are COMPLIMENTARY**

No measurements can simultaneously reveal the particle and the wave properties of matter

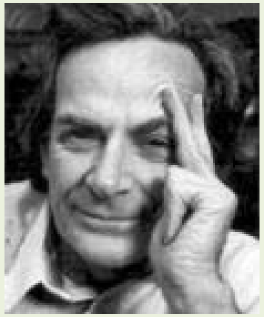
# Characteristic of Quantum mechanics

No general consensus

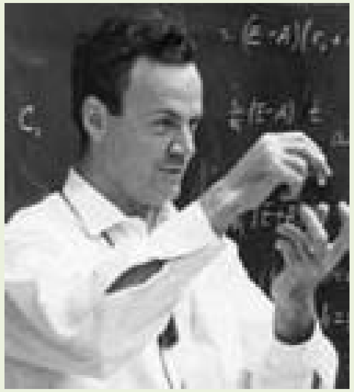
Can "do", but can't tell what we are doing.

Implausible

Niels Bohr: "If you are not confused by quantum physics then you haven't really understood it".



Richard Feynman: "I think I can safely say that nobody understands quantum mechanics".



# The Uncertainty Principle

**An experiment cannot simultaneously determine a component of the momentum of a particle (e.g., $p_x$) and the exact value of the corresponding coordinate, x.**

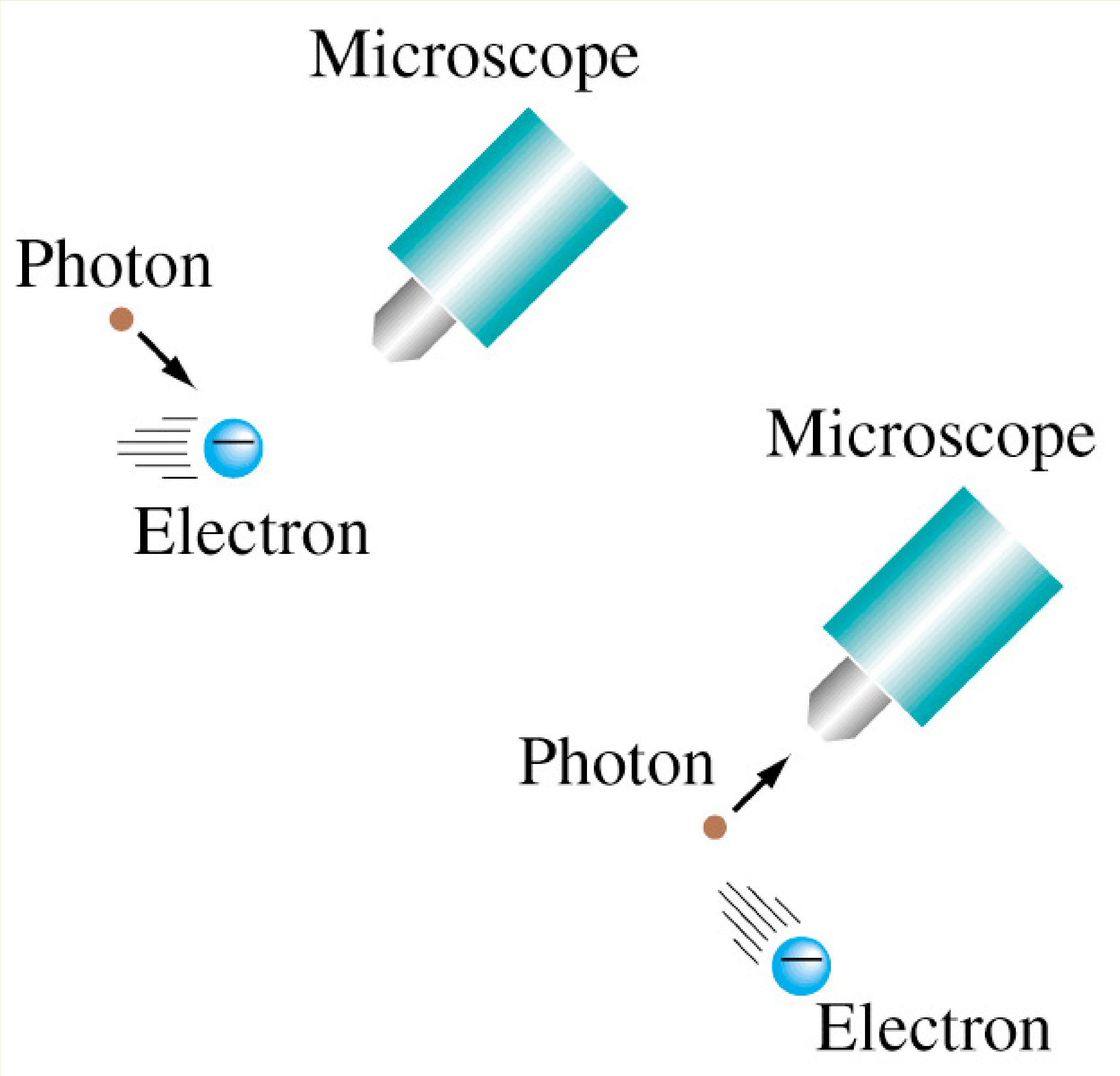
**The best one can do is**

$$(\Delta p_x)(\Delta x) \geq \frac{\hbar}{2}$$

# The Uncertainty Principle

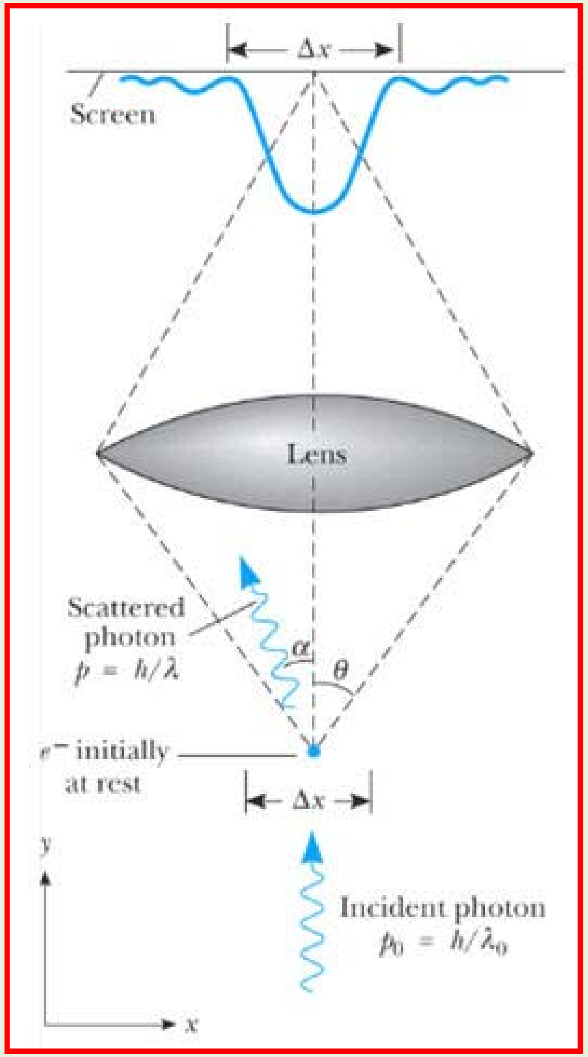
1. The limitations imposed by the uncertainty principle have nothing to do with quality of the experimental equipment
2. The uncertainty principle does imply that one cannot determine the position or the momentum with arbitrary accuracy
   - It refers to the impossibility of precise knowledge about both: e.g. if $\Delta x = 0$, then $\Delta p_x$ is infinity, and vice versa
3. The uncertainty principle is confirmed by experiment, and is a direct consequence of the de Broglie's hypothesis

# Heisenberg Microscope



# Physical Origin of the Uncertainty Principle
# Heisenberg (Bohr) Microscope

- The measurement itself introduces the uncertainty
- When we "look" at an object we see it via the photons that are detected by the microscope
  1. These are the photons that are scattered within an angle 2θ and collected by a lens of diameter *D*
  2. Momentum of electron is changed
  3. Consider single photon, this will introduce the minimum uncertainty



# Heisenberg (Bohr) Microscope

$$\Delta p_{ph}^{\max} = 2p_{ph}\sin\theta$$

As a consequence of momentum conservation

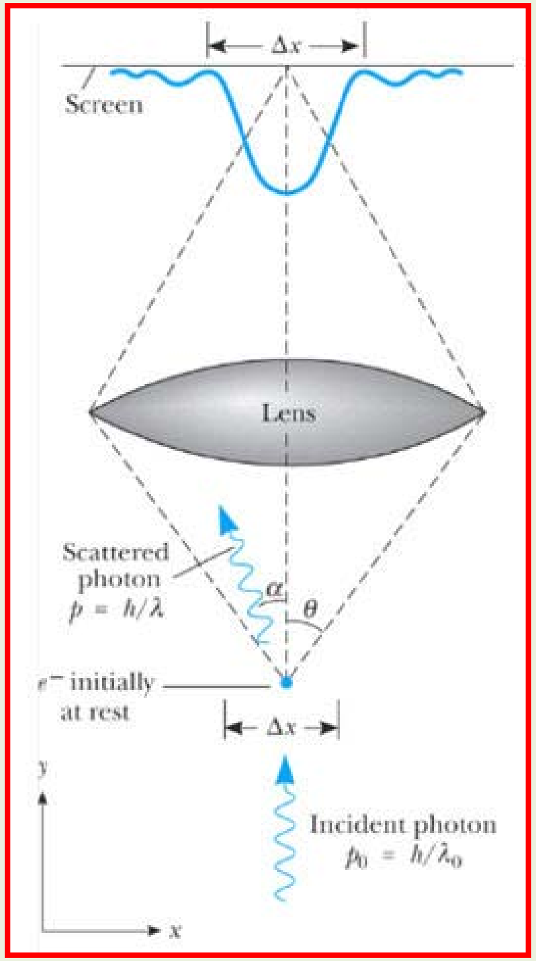
$$\Delta p_{electron} = \Delta p_{ph} = 2p_{ph}\sin\theta$$

$$p_{ph} = \frac{h}{\lambda}$$

$$\Delta p_{electron} = 2p_{ph}\sin\theta = \frac{2h}{\lambda}\sin\theta$$

$$\text{for small } \theta,\ \sin\theta \approx \theta$$

$$\Delta p_{electron} = \frac{2h}{\lambda}\theta$$




Trying to locate electron we introduce the uncertainty of the momentum

# Heisenberg (Bohr) Microscope

$$\Delta p_{electron} = \frac{2h\theta}{\lambda}$$

- $\theta$~(D/2)/L, L ~ D/2 $\theta$ is distance to lens
- Uncertainty in electron position for small $\theta$ is

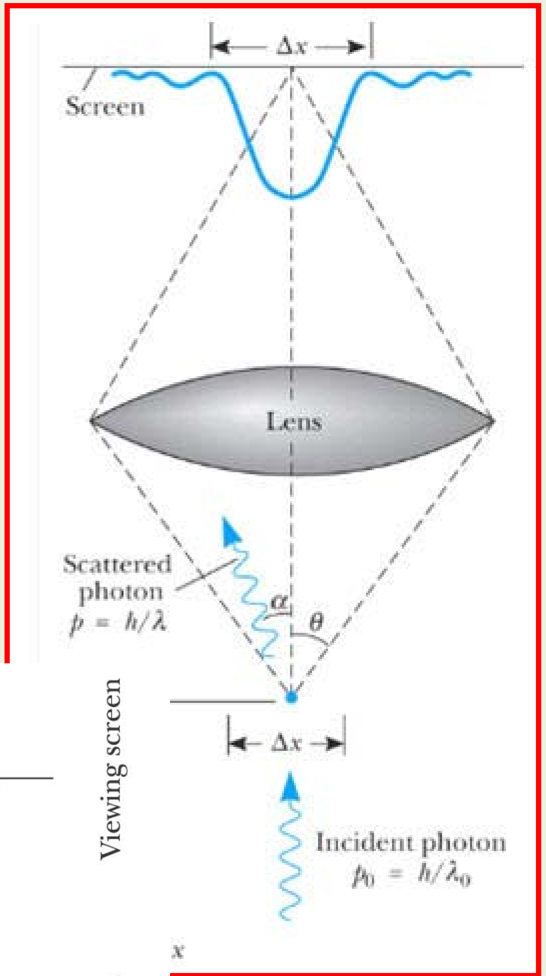
$$\Delta x_{electron} = L(2\theta) = (D/2\theta)(2\theta) = D$$

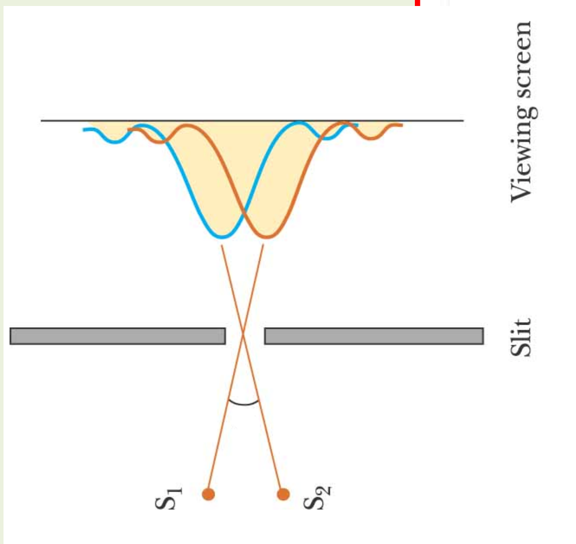
- To reduce uncertainty in the momentum, we can either increase the wavelength or reduce the angle
- But this leads to increased uncertainty in the position, since

$$\Delta x_{elelctron} = D$$

$$2\theta = 1.22\frac{\lambda}{D}$$

$$\Delta x_{electron} = \frac{1.22\lambda}{2\theta}$$



# Heisenberg (Bohr) Microscope

$$\left.\begin{aligned}\Delta p_{electron} &= \frac{2h}{\lambda}\theta \\ \theta &= \frac{1.22\lambda}{2D} \\ \Delta x &= D\end{aligned}\right\} \Rightarrow \Delta p_{electron} = 1.22\frac{h}{\Delta x}$$

$$(\Delta p_{electron})(\Delta x) = 1.22h$$

# The Uncertainty Principle

Between energy and time

$$(\Delta E)(\Delta t) \geq \frac{\hbar}{2}$$

# CONCLUSIONS

Light and matter exhibit wave-particle duality

Relation between wave and particle properties
given by the de Broglie relations

$$E = h\nu \qquad p = \frac{h}{\lambda}$$

Evidence for particle properties of light
Photoelectric effect, Compton scattering

Evidence for wave properties of matter
Electron diffraction, interference of matter waves
(electrons, neutrons, He atoms, C60 molecules)

Heisenberg uncertainty principle limits
simultaneous knowledge of conjugate variables

$$\Delta x \Delta p_x \geq \hbar / 2$$
$$\Delta y \Delta p_y \geq \hbar / 2$$
$$\Delta z \Delta p_z \geq \hbar / 2$$

# A SUMMARY OF DUAL ITY OF NATURE
## Wave particle duality of physical objects

LIGHT

Wave nature -EM wave

Optical microscope

Interference

Particle nature -photons

Convert light to electric curren

Photo-electric effect

PARTICLES

Wave nature

Matter waves
Electron microscope

Discrete (Quantum) states of confined systems, such as atoms.

Particle nature

Electric current
photon-electron collisions

- COMMON SENSE VIEW OF THE WORLD IS AN APPROXIMATION OF THE UNDERLYING BASIC QUANTUM DESCRIPTION OF OUR PHYSICAL WORLD !
- IN THE COPENHAGEN INTERPRETATION OF BOHR AND HEISENBERG IT IS IMPOSSIBLE IN PRINCIPLE FOR OUR WORLD TO BE DETERMINISTIC !
- EINSTEIN, A FOUNDER OF QM WAS UNCOMFORTABLE WITH THIS NTERPRETATION

# Bohr and Einstein in discussion 1933

# It's never as bad as it seems….


"Actually I started out in quantum mechanics, but somewhere along the way I took a wrong turn."